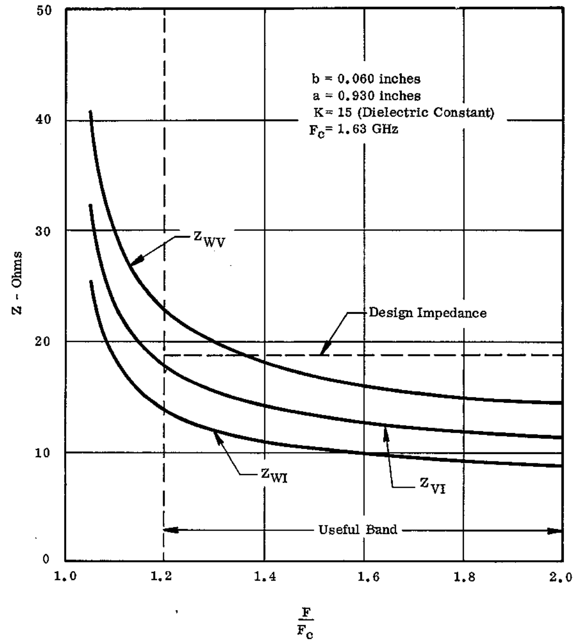

Fig. 2. Calculated waveguide impedance.

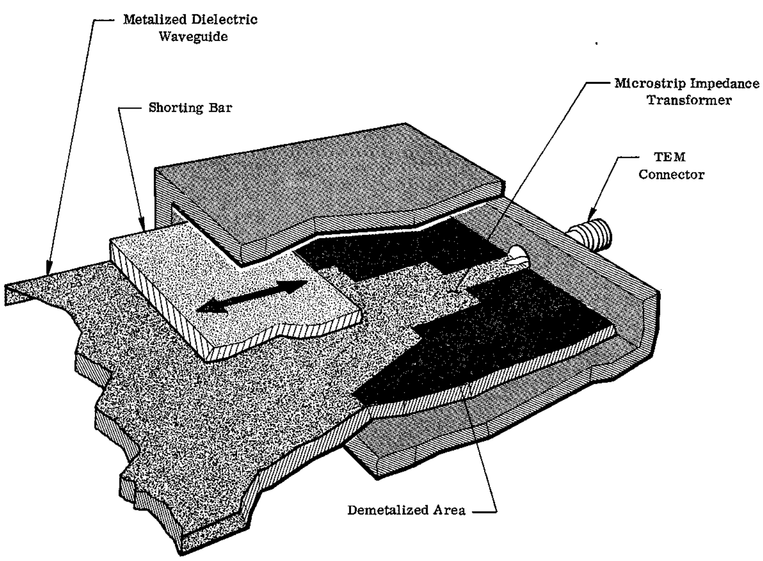

Fig. 3. TEM adapter cross section.

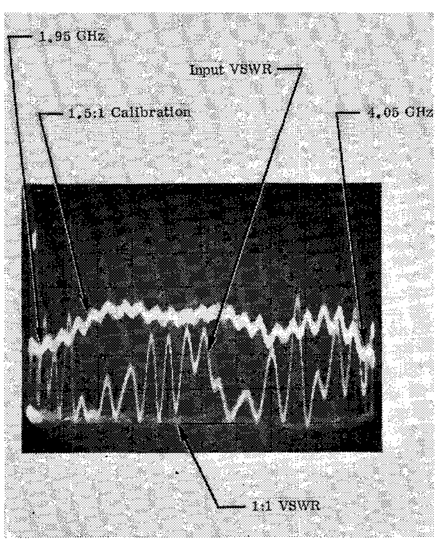

Fig. 4. Swept data input match adjusted adapter.

Acknowledgment

The efforts of G. F. Johnson who did the empirical optimizing of the transition are gratefully acknowledged.

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Guided Waves in Moving Media

In a recent paper,[1] the problem of electromagnetic wave propagation in a waveguide containing a moving dielectric medium was considered. It is shown in this correspondence that the approach used is unnecessarily complicated and that the mode behavior can be simply derived from the stationary problem by a Lorentz transformation to the frame of a moving observer.

The problem considered in Collier and Tai[1] was solved in the following manner: starting from Maxwell's equation and the constitutive equations in the axially-moving medium, the authors were able to derive vector and scalar potentials for both *E*- and *H*-type modes, satisfying a modified gauge condition. They were then able to write down a wave-type equation for the vector potentials which could be solved together with the boundary conditions at the waveguide walls. From this, the field components could be written down together with expressions for the wave impedance and axial propagation constant. These latter two were found to differ from their stationary values by a term independent of the waveguide dimensions.

In the present correspondence, it is shown that the problem may be solved in a much simpler way. The important point to note is that the movement of the medium relative to the waveguide is irrelevant to the solution of the problem. Since this movement is parallel to the axial velocity vector, the boundary conditions on the waveguide walls are exactly as in the stationary problem and the harmonic fields remain zero inside the waveguide walls. Physically, then, the problem is identical to that where the dielectric medium and waveguide move together along the z-axis at velocity $\bar{v}$ with respect to an observer, and the solution of this problem is related quite simply to the stationary problem (waveguide, medium, and observer all stationary) through a Lorentz transformation. In the stationary rectangular waveguide, a typical field component (unprimed system) is proportional to

$$\begin{matrix}\sin \\ \cos\end{matrix}\left(\frac{m\pi x}{a}\right)\begin{matrix}\sin \\ \cos\end{matrix}\left(\frac{n\pi y}{b}\right)\cdot \exp\left[i\Gamma_{mn}z - i\omega t\right] \tag{1}$$

where

$$\Gamma_{mn}^2 = \omega^2\mu\epsilon_{\text{eff}} - k_{mn}^2 = k^2 - k_{nm}^2$$

$$k_{mn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\epsilon_{\text{eff}} = \epsilon\left(1 + \frac{i\sigma}{\omega\epsilon}\right).$$

Manuscript received October 20, 1966.

[1] J. R. Collier, and C. T. Tai, *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-13, pp. 441–445, July 1965.

In the frame where the observer has an axial velocity v relative to the medium and waveguide (primed system), a typical field component is proportional to

$$\frac{\sin}{\cos}\left(\frac{m\pi x'}{a}\right)\frac{\sin}{\cos}\left(\frac{n\pi y'}{b}\right)$$

$$\exp\left[i\Gamma_{mn}'mn'z - i\omega' t'\right]$$

where the primed and unprimed coordinates are related through the Lorentz transformations

$$x' = x \quad y' = y \quad z' = \gamma(z + vt)$$

$$t' = \gamma\left(t + \frac{vz}{c_0^2}\right)$$

$$c_0^2 = \frac{1}{\mu_0\epsilon_0}$$

$$\exp[i\Gamma' z' - i\omega' t'] = \exp\left[i\gamma\left(\Gamma'_{mn} - \frac{\omega' v}{c_0}\right)z - i\gamma(\omega' - \Gamma'_{mn}v)t\right]$$

$$\exp\left[i\Gamma_{mn}z - i\omega t\right].$$

Therefore,

$$\Gamma_{mn} = \gamma\left(\Gamma'_{mn} - \frac{\omega' v}{c_0^2}\right) \quad \omega = \gamma(\omega' - \Gamma'_{mn}v)$$

Substituting in (2), we obtain, neglecting second-order terms in v,

$$\Gamma'_{mn} = +\omega' v\left(\frac{1}{c_0^2} - \mu\epsilon_{\text{eff}}\right) \pm (k^2 - k_{mn}^2)^{1/2}$$

$$= -\frac{\omega' v}{c^2}\left(1 - \frac{c^2}{c_0^2} + \frac{i\sigma}{\omega\epsilon}\right) \pm (k^2 - k_{mn}^2)^{1/2}$$

essentially equation (74) in Collier and Tai[1]

$$c^2 = \frac{1}{\mu\epsilon}.$$

The same procedure may be carried out for any cylindrical waveguide with similar results.

The field components and wave impedance follow directly from Maxwell's equations. The fact that the modification to the propagation constant and wave impedance is independent of the waveguide dimensions is therefore a direct consequence of the fact that the two problems are connected by a Lorentz transformation.

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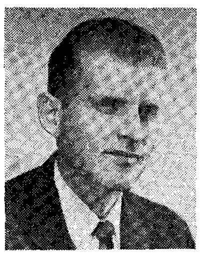

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